SECTION 8.2: INTEGRATION BY PARTS

RECALL: The Product Rule: $D_x\,[u(x)\,v(x)] = u'(x)\,v(x) + u(x)\,v'(x) = v(x)\,u'(x) + u(x)\,v'(x)$.

In differential form: $d\,[u(x)\,v(x)] = v(x)\,u'(x)\,dx = u(x)\,v'(x)\,dx$ or $d[u\,v] = v\,du + u\,dv$.

Hence, $u\,dv = d[u\,v] - v\,du$ so that: $\displaystyle\int u\,dv = \int [d[u\,v] - v\,du] = \int d[u\,v] - \int u\,dv = uv - \int v\,du$.

We've arrived at the famed 'Integration by Parts' formula (a reversal of the product rule for derivatives!)

INTEGRATION BY PARTS: $\displaystyle\int u\,dv = uv - \int v\,du$.

STRATEGY: Make a choice for 'u' and what's left is 'dv' or recognize a 'dv' and what's left over is 'u.'

RULE OF THUMB: Your choice for 'u' should make the integrand somewhat easier when differentiating.

With that in mind, keep in mind the acronym: 'LIATE':

Let 'u' be the: L(og) or I(nverse), A(lgebraic), T(rig) or E(xponential) factor in the integrand.

USING INTEGRATION BY PARTS:

1. Make a choice for 'u.'
2. Differentiate 'u' to obtain du and integrate 'dv' to obtain v.
3. Is the integral $\displaystyle\int v\,du$ in any way 'easier' to handle then the original integral?
4. If so, proceed! If not, re-evaluate your choice for 'u.'
5. Repeat as needed.

NOTE: Before trying parts, make sure you've rule out the other more basic approaches.

EXAMPLE 1: Find the following integrals. Check your answers.

1. $\displaystyle\int x\,e^{3x}\,dx$ — Ans: $\dfrac{1}{3}\,x\,e^{3x} - \dfrac{1}{9}\,e^{3x} + C$
2. $\displaystyle\int x^2\,e^{3x}\,dx$ — Ans: $\dfrac{1}{3}\,x^2\,e^{3x} - \dfrac{2}{9}\,x\,e^{3x} + \dfrac{2}{27}\,e^{3x} + C$
3. $\displaystyle\int \tan^{-1}(x)\,dx$ — Ans: $x\,\tan^{-1}(x) - \dfrac{1}{2}\ln\left(x^2+1\right) + C$
4. $\displaystyle\int_0^{\frac{\pi}{2}} t\,\cos(2t)\,dt$ — Ans: $-\dfrac{1}{2}$

EXAMPLE 2: (VIDEO) Refer to Example 4 in Section 8.2 to help you find the following integrals.

1. $\displaystyle\int e^{-x}\cos(x)\,dx$ Ans: $\dfrac{1}{2}e^{-x}(\sin(x)-\cos(x))+C$

2. $\displaystyle\int \sec^3(\theta)\,d\theta$ Ans: $\dfrac{1}{2}(\sec(\theta)\tan(\theta)+\ln|\sec(\theta)+\tan(\theta)|)+C$

EXAMPLE 3: (VIDEO) Use Integration by Parts to verify the following Reduction Formula:

$$\int \cos^n(x)\,dx=\frac{1}{n}\cos^{n-1}(x)\sin(x)+\frac{n-1}{n}\int\cos^{n-2}(x)\,dx$$

EXAMPLE 4: (VIDEO) Use Tabular Integration (Szoke's Quickie Method) to find: $\displaystyle\int t^3\sin(2t)\,dt$.

Ans: $-\dfrac{1}{2}t^3\cos(2t)+\dfrac{3}{4}t^2\sin(2t)+\dfrac{3}{4}t\cos(2t)-\dfrac{3}{8}\sin(2t)+C$

EXAMPLE 5: (VIDEO) Use Integration by Parts to verify the formula: If $s\neq 0$:

$$\int_a^b f'(t)e^{-st}\,dt=f(t)e^{-st}\Big|_{t=a}^{t=b}+s\int_a^b f(t)\,e^{-st}\,dt$$

HOMEWORK: Section 8.2: 9 - 41 odd, 51 and 55, 53 and 57, 59